Ion channels are embedded in the plasma membrane, a compositionally diverse two-dimensional liquid that has the potential to exert profound influence on their function. Recent experiments suggest that this membrane is poised close to an Ising critical point, below which cell-derived plasma membrane vesicles phase separate into coexisting liquid phases. Related critical points have long been the focus of study in simplified physical systems, but their potential roles in biological function have been underexplored. Here we apply both exact and stochastic techniques to the lattice Ising model to study several ramifications of proximity to criticality for idealized lattice channels, whose function is coupled through boundary interactions to critical fluctuations of membrane composition. Because of diverging susceptibilities of system properties to thermodynamic parameters near a critical point, such a lattice channel’s activity becomes strongly influenced by perturbations that affect the critical temperature of the underlying Ising model. In addition, its kinetics acquire a range of time scales from its surrounding membrane, naturally leading to non-Markovian dynamics. Our model may help to unify existing experimental results relating the effects of small-molecule perturbations on membrane properties and ion channel function. We also suggest ways in which the role of this mechanism in regulating real ion channels and other membrane-bound proteins could be tested in the future.

Introduction

In addition to separating the cell from its surroundings, the plasma membrane is home to diverse functional processes. Membrane-bound ion channels sense chemical and electrical signals and control conductance to specific ions, leading to the complex dynamics that underlie neural function. Although most ion channels are broadly classified as ligand gated or voltage gated, many are also sensitive to a wide range of modulators including calcium levels, pH, temperature, and lipids (Kinnunen, 1991). Ion channel function can depend on the 2-D solvent properties of the membrane in which they are embedded, in both reconstituted (Bristow and Martin, 1987; Rankin et al., 1997) and in vivo assays (Sooksawate and Simmonds, 2001; Allen et al., 2007). Furthermore, structural studies have demonstrated close association between particular lipids and ion channels (Barrantes, 2004; Zhu et al., 2018), sometimes dependent on their functional state (Gao et al., 2016).

Although early efforts assumed the membrane to be a homogenous 2-D solvent for embedded proteins, it is now thought that the membrane is heterogeneous, with liquid structures often termed “rafts” at length scales of 10–100 nm, much larger than the 1-nm scale of individual lipids (Simons and Toomre, 2000; Dart, 2010). Neurotransmitter receptors are often found to be associated with ordered membrane domains when probed with biochemical methods (Allen et al., 2007), sometimes in a subtype-specific manner (Li et al., 2007). Many channels and channel scaffolding elements are posttranslationally lipitated with palmitoyl groups (Fukata and Fukata, 2010; Fukata et al., 2013; Borroni et al., 2016; Globo and Banjji, 2017), and mutation of palmitoylated cysteines reduces their localization to synapses (Christopherson et al., 2003; Raathenberg et al., 2004; Delint-Ramirez et al., 2011). Protein palmitoylation is highly correlated with partitioning into ordered membrane domains in both intact and isolated biological membranes (Levental et al., 2010; Lorent et al., 2018).

Experiments have suggested a physical mechanism that may underlie these structures. Vesicles isolated from mammalian cell lines have membranes tuned close to a liquid–liquid miscibility critical point (Veatch et al., 2008). When cooled below their critical temperature, $T_c$, these vesicles macroscopically phase separate into two liquid phases termed liquid ordered ($L_{o}$) and liquid disordered ($L_{d}$), which differ in the partitioning of lipids, proteins, and a fluorescent dye (Baumgart et al., 2007). Experiments...
suggest that in physiological conditions, plasma membranes exist close to but above a miscibility criticality point. In this nearly critical region, they are expected to display a range of critical phenomena that arise because the free energy cost of making relatively large domains becomes comparable to the thermal energy. Components of a fluid membrane are expected to exhibit long-range correlations in space, with a correlation length $\xi$ that diverges as the critical point is approached, $\xi \sim (T - T_c)^{-\nu}$ (Cardy, 1996). Critical systems also have a long memory, with a correlation time $\tau$ that diverges as the critical point is approached, $\tau \sim \xi^z$ with $z$ a dynamic critical exponent (Hohenberg and Halperin, 1977; Honerkamp-Smith et al., 2012). In addition, changing control parameters such as temperature and the addition of small molecules to the chemical environment leads to large changes to system properties. The influence of a small change in system parameters on physical properties (“susceptibility”) diverges as the critical point is approached, leaving membrane properties particularly sensitive to external perturbations in their critical region. Systems near criticality exhibit “universal” properties that are largely independent of the microscopic properties of the particular system under consideration, making them highly amenable to quantitative description with highly simplified models, a property we take advantage of here.

Previously, we have argued that proximity to this critical point is likely to underlie much of the raft heterogeneity seen in diverse membrane systems (Machta et al., 2011), with embedded proteins subject to long-range critical Casimir forces (Machta et al., 2012). More recently, we have shown that n-alcohol anesthetics take membrane-derived vesicles away from criticality by lowering $T_c$ (Gray et al., 2013). Despite structural diversity, n-alcohol anesthetics are known to exert similar effects on diverse ion channels (Franks and Lieb, 1994), leading us to speculate that alcohol anesthetics are known to exert similar effects on diverse hydrophobic regions of ion channels (Mihic et al., 1997; Borghese et al., 2006; Nury et al., 2011; LeBard et al., 2012; Yip et al., 2013), which are in the model B universality class (Hohenberg and Halperin, 1977). All dynamic simulations were performed on $128 \times 128$ lattices with temperature and lattice channel boundary conditions as described in Results. We define a sweep to be a time step in which $128^2$ moves are attempted, so that every spin is attempted to move on average twice. Every sweep, we supplement these dynamics with a single attempt to flip the lattice channel to satisfy detailed balance according to the Metropolis probability.

Online supplemental material
In the supplemental material, we first provide a more detailed argument for the form of the scaling function needed to capture fixed/free boundary conditions used to make Fig. 2 D. We then discuss the scaling form of the dynamic correlation function $\chi(\tau, r, \xi)$ and the effect of our choice of $J_R \to -\infty$. Finally, we discuss subtleties in our Gaussian blurring procedure used to make Fig. 3 A. Figure S1 shows that results are robust to changes in $J_R$. Figure S2 shows results are robust to resolution of Gaussian blurring.

Results and discussion
We consider a lattice system consisting of membrane degrees of freedom $\{s\}$ and a single hypothetical lattice channel (Fig. 1 A) with state $R$ and Hamiltonian $\mathcal{H}_{\text{tot}}(\{s\}, R)$. The Hamiltonian of our combined system comprises three components:

$$\mathcal{H}_{\text{tot}}(\{s\}, R) = \mathcal{H}_{\text{mem}}(\{s\}) + E_R + \mathcal{H}_{\text{int}}(\{s\}, R),$$  \hspace{1cm} (i)

where $\mathcal{H}_{\text{mem}}$ describes interactions of membrane lipids with one another, $E_R$ measures the energy of the lattice channel in
that can take two values, R = ±1, which could correspond to open/closed. This state can be affected by a chemical potential (perhaps a ligand) that in our model contributes a term in the Hamiltonian ER = −µR. Crucially, we also include an interaction between a protein's fixed interaction with its surrounding membrane to understand the membrane-mediated critical Casimir forces that act on a pair of proteins (Machta et al., 2012). Previous authors have also considered the role of single-site proteins in an Ising model which are driven out of equilibrium and which thereby prevent phase separation (Sabra and Mouritsen, 1998). To understand how criticality could directly influence protein conformational equilibria, we consider three types of boundary conditions: fixed boundary conditions in which hi(R) = ±JR, free boundary conditions in which hi(R) = 0, and Janus boundary conditions in which hi(R) = ±JR on one side of the lattice channel and hi(R) = −JR on the other side. We consider these simplest boundary conditions to gain a qualitative understanding of the effects of different boundary conditions on the lattice channel's static properties.

In previous work, we have considered a similar model for a protein’s fixed interaction with its surrounding membrane to understand the membrane-mediated critical Casimir forces that act on a pair of proteins (Machta et al., 2012). Previous authors have also considered the role of single-site proteins in an Ising model which are driven out of equilibrium and which thereby prevent phase separation (Sabra and Mouritsen, 1998). To understand how criticality could directly influence protein conformational equilibria, we consider three types of boundary conditions: fixed boundary conditions in which hi(R) = ±JR, free boundary conditions in which hi(R) = 0, and Janus boundary conditions in which hi(R) = ±JR on one side of the lattice channel and hi(R) = −JR on the other side. We consider these simplest boundary conditions to gain a qualitative understanding of the effects of different boundary conditions on the lattice channel's static properties.

For our studies of mean activity, we use JR → ∞ for consistency with our past work (Machta et al., 2012), corresponding directly to a physical system in which a lattice channel of slightly larger radius has interaction energy hi = J. In the supplemental text, we explore the dependence of our results on the strength of the coupling Jk. We find that the large interaction energies we use will give quantitatively similar results to a system in which a lattice channel of the same radius has interactions at least on the order of k_BT.

### Lattice channel average conductance

We first investigated how a change in the membrane’s properties might influence average channel activity, as would be measured by a whole-cell recording or other technique probing the response of many channels together. In our model, the membrane and the lattice channel embedded within it are in thermal equilibrium. Membrane (Ising spin) degrees of freedom are tuned close to a liquid–liquid miscibility critical point, and a single lattice channel interacts with the membrane by preferring different local liquid environments when in different functional states. In equilibrium, the system consisting of lattice channel and membrane will be in a given state with probability determined by an appropriate Boltzmann distribution,

\[
P((s), R) = e^{-\beta\mathcal{F}_\text{mem}(s), R}/Z,
\]

where Z is defined such that the sum of probabilities is unity.

We can integrate over membrane degrees of freedom to isolate the lattice channel whose internal states are occupied according to P(R) = \( e^{-\beta\mathcal{F}_\text{tot}(R)}/Z \), where
\[
\exp(-\beta \mathcal{F}_R) = \sum_i \exp[-\beta \mathcal{H}_\text{mem}(i, R)]
\]  

(2)

defines \( \mathcal{F}_R \), the free energy of state \( R \).

In the context of our model, changes to membrane miscibility influence lattice channel activity if \( P(R) \) depends on the details of \( \mathcal{H}_\text{mem} \). Changes to \( \mathcal{H}_\text{mem} \) can lead to a change in a state’s free energy \( \Delta \mathcal{F}_R \), with resulting changes in \( P(R) \). In this two-state example, the membrane influences the activity of the lattice channel, \( P(R = \pm 1) \), through changes to \( \Delta \mathcal{F}_R \). In the case where \( P(R = \pm 1) \ll 1 \), \( P(R = \pm 1) \) increases by a factor of \( \exp(-\beta \Delta \mathcal{F}_R) \).

In the general case, the ratio \( P(R = +1)/P(R = -1) \) increases by \( \exp(-\beta \Delta \mathcal{F}_\text{diff}) \). Thus, \( \exp(-\beta \Delta \mathcal{F}_\text{diff}) \) measures the degree to which a change in \( \mathcal{H}_\text{mem} \) potentiates the + state (Fig. 2A).

It is often difficult to estimate free energies from simulation, but for 2-D lattice systems, there has been substantial recent progress (Thomas and Middleton, 2013), which we can use for our lattice membrane/channel system. To study the static properties of our equilibrium system, we make use of a remarkable algorithm developed for spin glasses that uses Pfaffian elimination to exactly calculate partition functions to user-specified precision on lattices with arbitrary nearest neighbor couplings \( J_{ij} \) in zero field (Thomas and Middleton, 2013). We implement the Hamiltonian defined previously by setting all interactions \( J_{ij} \) between spins to \( \beta \), interactions between lattice sites internal to the lattice channel to minus infinity, and interactions between lattice sites contained in the lattice channel and spins to \( h_i(R) \) (Fig. 1B).

We estimate the potentiation of function that accompanies a 1% change in \( T_c \), comparable both to the natural variation seen in cell-derived vesicles (Veatch et al., 2008) and to the magnitude of the effect seen with physiologically relevant concentrations of n-alcohols (Gray et al., 2013). We examine boundary conditions in which the \( R = -1 \) state has fixed boundary conditions (\( h_i = +\infty \)), whereas the \( R = +1 \) state has either free boundary conditions (\( h_i = 0 \)) or Janus boundary conditions. The results are shown in Fig. 2 (B–D). In Fig. 2B, this potentiation is shown for lattice channels in which the \( R = +1 \) state has free boundary conditions, at \( T = 1.05 T_c \), for channels with a range of radii (the lattice channels are displayed in Fig. 4 and in the supplemental material, and the radii are calculated as the radius of the circle circumscribing each lattice channel). Larger lattice channels see more pronounced potentiation owing to the increased boundary length of interaction. In Fig. 2C, the potentiation is shown for the same lattice channels over a range of temperatures. Away from the critical point, this change has only a small effect on the state occupancy—the potentiation is small. However, near the critical point, the potentiation becomes much larger, even for the modest 1% change in \( T_c \), explored here. This change in \( T_c \) is comparable to that seen with addition of clinically relevant concentrations of anesthetic (Gray et al., 2013; Machta et al., 2016). For the largest lattice channel sizes explored here, our observed potentiation is similar to that observed for the GABA\(_A\) channel (see Fig. 2 of Franks and Lieb [1994]) whose radius is \( \sim 6 \) nm. Although we do not know details of GABA\(_A\)’s interaction with its surrounding membrane, our results imply that anesthetic effects on membrane \( T_c \) could in principle lead to observed changes in channel function even in the absence of direct binding interactions between compound and channel.

An Ising lattice at fixed chemical potential for up spins (not included in our model) has a diverging susceptibility; a small

For channels regulated through this mechanism, we expect that, for a given channel and given boundary conditions, different perturbations should have effects on channel activity predicted solely through that perturbation’s influence on \( T_c \). Experimental results on three different channels that respond differently to n-alcohols are consistent with this prediction. In Fig. 3, we plot results from whole-cell channel recordings from Mascia et al. (1996), Nakahiro et al. (1996), and Zuo et al. (2001) for several channels in the presence of n-alcohols plotted against extrapolated \( \Delta T_c \) as investigated in Gray et al. (2013). As shown in the figure, for any given channel, the effects of these n-alcohols are well predicted by their effect on \( T_c \). Although this collapse of the data is certainly suggestive, future work is needed to exclude a more mundane possibility: that it arises because of a mutual correlation of both assays with the hydrophobicity of different n-alcohols.
change in this chemical potential leads to a diverging change in the number of up spins near the critical point, which would result in large changes to the activity of our lattice channel. However, for most components, biological membranes are held at fixed particle number rather than chemical potential, equivalent to fixed number (not chemical potential) of up spins in our model. Changes to the number of up spins, as would accompany biological changes in most lipid levels, do not lead to large changes to the lattice channel's activity. One exception to this could be cholesterol which can be rapidly transferred between internal membranes and the plasma membrane so that its plasma membrane concentration under perturbation is fully described by neither a fixed chemical potential nor a fixed particle number. More broadly, the relationship between particle number and chemical potential requires care to interpret (Ayuyan and Cohen, 2018).

**Lattice channel kinetics**

We next looked at the role a critical membrane could play in shaping the kinetics of ion channel function as would be measured in single-channel voltage clamp experiments. Real ion channels have kinetics that span a wide array of time scales that are typically understood as arising from internal states of the channel itself (Mortensen and Smart, 2007). Here we consider a hypothetical lattice channel that contains only a single time scale, but that couples to its surrounding lipids, acquiring slower time scales from the critical membrane in which it is embedded. In our studies of dynamics, we consider fixed boundary conditions and use $h_i(R) = R_j$, the same as the interaction between neighboring lipids ($R = \pm 1$ is the state of the lattice channel).

We set $E_k = 0$ to treat both states of the lattice channel symmetrically. We use the local Kawasaki algorithm (Kawasaki, 1972) as we have done previously (Machta et al., 2011) which implements diffusive model B dynamics (Hohenberg and Halperin, 1977). This algorithm exchanges neighboring spins with probabilities chosen to maintain detailed balance. Using these dynamics, a sweep (in which every spin is proposed to exchange with its neighbors once) corresponds to roughly a microsecond of real time, because each lattice site corresponds to the size of a lipid (~1 nm$^2$) and lipids diffuse at a rate of ~1 μm$^2$/s (Machta et al., 2011). One caveat is that cells are likely in the regime where hydrodynamic relaxation (Honerkamp-Smith et al., 2012) competes with diffusive dynamics. Our Kawasaki dynamics forbid exchange with spins in the lattice channel, but every sweep the lattice channel changes state with the Metropolis probability to ensure detailed balance.

Although our lattice channel attempts to change its state $R$ during every sweep, its apparent kinetics are much slower. Representative traces of $R(\tau)$ are shown in Fig. 4A after convolution with a Gaussian of width $\sigma = 10^3$ sweeps. Because patch-clamp recordings measure times on the order of 50–100 μs (Hamill et al., 1981; Kasianowicz et al., 1996; Sakmann and Neher, 2009), we plot $R(\tau)$ after convolution with the Gaussian $k(\tau) = e^{-\tau^2/\sigma^2}$, where $\sigma = 10^3$ (see supplemental material for bare curves and a discussion). Thus, although our lattice channel has only two states, the recording shows short spikes reaching levels in between the two, a feature common to real recordings that presumably has a similar explanation (Hamill et al., 1981). The traces shown in Fig. 4A share qualitative features with real ion channel traces, spanning many time scales (Mortensen and Smart, 2007): “flickers” shorter than $\sigma$, single “openings” (which often contain many short flickers), and “bursts” in which a series of openings occur close to each other. The time scale of these features increases as the critical point is approached (Fig. 4A). Similar changes in the time scale of ion channel dynamics arising from the addition of some anesthetics have been observed experimentally (Wachtel, 1995). Our findings indicate that these could result from perturbations to the proximity of the membrane to criticality, although they are typically interpreted as arising from direct binding interactions between anesthetic and channel. It is interesting to note that synthetic membranes brought close to a possibly analogous liquid–gel critical point can display currents in the absence of proteins that bear a striking resemblance to those mediated by ion channels (Antonov et al., 1980; Wodzinska et al., 2009). Our results suggest that this commonality may arise because both protein channel and membrane-mediated currents acquire their long time scales from critical fluctuations of the membrane itself.

We quantify these findings by looking at the time autocorrelation of the lattice channel, $\chi(\tau) = \langle R(0)R(\tau) \rangle$ for a range of temperatures (Fig. 4B), demonstrating that correlations in state persist much longer as the critical point is approached. We also perform simulations for lattice channels of different radii, all at $T = 1.05T_c$ (Fig. 4C), demonstrating that larger lattice channels have slower kinetics owing to the increased area for interaction.
size effects were found to be negligible by performing similar studies on lact-

Regulation by a critical membrane

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behavior of several real ion channels and other membrane-em-

dynamics that are highly non-Markovian. The non-Markovian

scales from the membrane in which they are embedded and have

the critical point, the lattice channels acquire a range of time

sweeps. Simulations were performed on a lattice of \( N = 128 \times 128 \), and finite

T_c, lattice channel state changes occur on a time scale orders of magnitude

larger than that of attempted state changes, which happen once per sweep.

(B) The autocorrelation function of \( \chi(\tau) \) is shown at different temperatures,

with proximity to criticality leading to longer-lived correlations. Deviation

of \( \chi \) from a simple exponential, which would be a straight line in the inset,
suggests that protein dynamics occur on many time scales, implying that

although the system as a whole is Markovian, the protein considered in isol-

ation is non-Markovian. (C) \( \chi(\tau) \) also depends on lattice channel radius \( r \)

at a single temperature, \( T = 1.05 T_c \), (colors as above). (D) These long time scales

are critical phenomena. When we plot \( \chi(\tau)/r \) versus \( \tau/\xi \) for \( r \) and \( \xi \) values

chosen so that \( r/\xi \) is constant, these curves collapse onto a single universal function as predicted by scaling. Each curve is calculated from data from \( 10^4 \) sweeps. Simulations were performed on a lattice of \( N = 128 \times 128 \), and finite

size effects were found to be negligible by performing similar studies on latt-

tices of four times the area.

with surrounding membrane. These curves quantify the many
time scales that can be seen qualitatively in Fig. 4 A; they are not closely approximated by single exponentials.

Although the combined system of lattice channel and mem-
brane taken together is Markovian, when considered in isola-
tion, the lattice channel displays non-Markovian dynamics, as
evidenced by the nonexponential autocorrelation function. This
implies that some memory of its history is stored in the mem-
brane degrees of freedom surrounding it. Far from criticality,
where no long time scales are inherent to the membrane, this
effect is small, and lattice channel kinetics, even when consid-
ered in isolation, appear almost Markovian. However, close to
the critical point, the lattice channels acquire a range of time
scales from the membrane in which they are embedded and have
dynamics that are highly non-Markovian. The non-Markovian
behavior of several real ion channels and other membrane-em-

bedded proteins has been well characterized, and various in-
ternal states of these proteins have been hypothesized (Rankin et al., 1997; Yamashita et al., 2005; Schmauder et al., 2011). It is
certainly likely that much of this observed non-Markovian be-

havior does indeed arise because these channels contain internal
states not directly probed with conductance. However, our re-
sults suggest that, in some cases, the history of the protein’s state
may be stored in the membrane rather than in internal states of
the protein itself.

Scaling collapse of average conductance

We next wanted to verify that our observed effects could be
understood as a critical phenomenon. Properties that arise
due to proximity to criticality are often universal, applying
to surprisingly diverse real systems as well as simplified
minimal models. Scaling functions are a useful tool to probe

universality. These functions only describe the universal prop-
erties of a system and do not contain the specific details that
distinguish different real systems, or even real systems from
models within the same universality class. As a result, scaling
collapses are important tests of which model predictions are
expected to apply to real systems that are qualitatively differ-

ent in their microscopic details. According to scaling, the free
energy associated with inserting a lattice channel with partic-

ular boundary conditions should have a universal contribution
that depends on lattice channel size, boundary conditions, and

the distance to the critical point:

\[
\mathcal{F}_R = U_1(r t^{1/\nu}, h t^{\phi/\nu},...),
\]

where \( y_t = \nu = 1 \) and \( y_h = \gamma = 15/8 \) are the scaling dimension of the

fields \( t \) and \( h \), \( r \) is the radius of the lattice channel, and \( U_1 \) is a uni-

versal function. Eq. 3 represents our knowledge of the functional
form of \( \mathcal{F}_R \); although we do not know the specifics of its depen-
dence on \( t, r, \) and \( h \), we know from scaling that it does not depend

on each independently, but rather on the combinations \( r t^{1/\nu} \) and \( h t^{\phi/\nu} \), as well as on other parameters represented by the ellipsis.

Taking a derivative of Eq. 3 with respect to \( t \), we find that

\[
\frac{\partial \mathcal{F}_R}{\partial t} = r^{\nu} U_2(r t^{1/\nu}, h / t^{\phi/\nu},...),
\]

where \( U_2 \) is another universal function that is an appropriate

sum of derivatives of \( U_1 \).

We verify that our measured potentiation can indeed be col-
lapsed in the manner predicted by Eq. 4 in Fig. 2 D for the fixed/
Janus boundary conditions. However, for the fixed/free systems,
we find an additional contribution that arises more directly
from the boundary. This term is proportional to the length of
the boundary and should scale as

\[
\mathcal{F}_{R,2} = r l U_3(l t^{1/\nu}, h / t^{\phi/\nu},...),
\]

leading to a similar collapse but with \( l t^{1/\nu} \) on the \( x \) axis (see supple-
mental material for discussion). Because the microscopics of our
model do not enter into the derivations of the above equations,
the collapse to these universal functions depicted in Fig. 2 for our
simulations demonstrates that it is the proximity to criticality of
our simulations, rather than their microscopic details, that dom-
inate our results.
Scaling collapse of lattice channel kinetics
We expect that the long time scales seen in Fig. 4 A are inherited from the surrounding membrane. Near the Ising critical point, the correlation length diverges as \( \xi \propto t^{-\eta} \) (where \( t = (T - T_c)/T_c \)), and the correlation time diverges as \( \tau_{\text{cor}} \propto \xi^z \) where a dynamic exponent is \( z = 4 - \eta = 3.75 \) for the Kawasaki dynamics used here (Hohenberg and Halperin, 1977). If the curves of \( R(\tau) \) reflect properties of these slow critical fluctuations, then we might expect that autocorrelation functions in systems with different radii and correlation lengths \( \chi(\tau, r, \xi) \) might take a universal form:

\[
\chi(\tau, r, \xi) = \lambda \mathcal{U}(r/\xi, \tau/\xi),
\]

with \( \lambda = 2y_h = 1 \) where \( y_h \) is the scaling dimension of the boundary operator, which is expected to be 1/2 from conformal field theory arguments (Cardy, 2004; see supplemental material for a brief discussion). To check this, we performed simulations for the same lattice channels shown in Fig. 4 C but now at different temperatures chosen so that \( r/\xi \) is fixed. Although the bare curves decay over time scales that differ by a factor of \( 10^2 \), when we plot \( \chi(\tau)/r \) versus \( \tau/\xi^z \), we see collapse onto a single curve within stochastic error (Fig. 4 D) as predicted by Eq. 6, thus demonstrating that the dynamics observed are critical phenomena.

Conclusions
Regulation by the nearly critical membrane might be very widespread for both ion channels and other membrane-bound proteins. There are hints that the membrane may be playing an important role in localization (Li et al., 2007) and in some cases in direct regulation (Allen et al., 2007) of channels. Our model could account for the sensitivity of many diverse ion channels to chemically diverse anesthetics (Franks and Lieb, 1994), which we have demonstrated lower the critical temperature of cell-derived vesicles by \( \sim 4 \text{°K} \), just over 1% (Gray et al., 2013). Although this model remains to be critically tested, this work suggests that it is thermodynamically feasible for n-alcohols and other hydrophobic components to influence channel function without directly binding to channels. This explanation is appealing in that it might also explain why cholesterol depletion, which changes the membrane’s “magnetization” (Zhao et al., 2013), also affects many anesthetic-sensitive channels (Sooksawate and Simmonds, 2001). Recent lipidomics work in neuronal and synthetically derived vesicles have shown that membrane composition undergoes regulated changes during development (Tulodziecka et al., 2016), possibly in part to modulate channel function through a mechanism like the one explored here.

Recent work has demonstrated that anesthetics have little influence on the mean conductance of gramicidin channels, whose gating is quite sensitive to some combination of features of their local membrane environment (Herold et al., 2017). Although those authors interpret this as evidence that anesthetics do not influence membrane properties, our results naturally predict that gramicidin, a small peptide, should be relatively insensitive to changes in \( T_c \). Gramicidin channels dimerize across leaflets, forming a pore and leading to a deformation of the surrounding membrane. The energetic cost of this deformation depends strongly on the hydrophobic thickness of the membrane, leading to dependence of function on some membrane properties. Our work suggests that these relatively small channels would not be particularly sensitive to perturbations that only influence \( T_c \). Perturbations that influence \( T_c \) alone do not change the local structure of the membrane dramatically, but instead influence the sizes of typical domains. As such, channels sensitive to \( T_c \) must be relatively large. Gramicidin takes up only a slightly larger area than a lipid and is therefore much smaller than anesthetic-sensitive channels, which tend to be \( \sim 10 \text{nm} \) across.

In Weinrich et al. (2017), the authors show that when the membrane is in a single phase state, gramicidin channels have kinetics described by a single exponential for the channel dwell time, consistent with the predictions of our model in the absence of membrane coupling. However, in membranes with two coexisting phases, the channel adopts kinetics that cannot be fitted with a single exponential, but are reasonably well described by a fit to the sum of two exponentials. The authors left open the possibility that the two time scales come from different kinetics in the two phases, or from a mechanism where channels acquired more complex kinetics from switching between the phases. The latter is similar to our model.

Our results suggest that channels modulated in the manner considered here should have their partitioning into small domains modulated by the addition of ligand, a prediction that could be tested using superresolution techniques. In cells, partitioning into larger domains below the critical temperature is likely to be complicated by interactions with the cortical cytoskeleton, which interfere with macroscopic phase separation (Machta et al., 2011). Although macroscopic domains are observed in plasma membrane vesicles isolated from cortical cytoskeleton and viewed at low temperature, it is not known whether these membranes preserve the physical and chemical properties of domains important for channel regulation by this mechanism. Although most previous studies, both by others (Reynwar and Deserno, 2008; Katira et al., 2016) and by ourselves (Machta et al., 2011, 2012), have focused on the role of membrane thermodynamics in localizing proteins into domains, this work suggests that the same domains could couple more directly to function. We predict that when a receptor binds ligand, it will change its preference for its surrounding lipid environment, leading to different interaction partners and an imprint on the state of the local membrane that will persist even after the ligand has dispersed. We have highlighted a specific impact of thermodynamic criticality on a single ion channel, and further work will clarify how this mechanism contributes to neural function.

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