A theoretical model of cytokinesis implicates feedback between membrane curvature and cytoskeletal organization in asymmetric cytokinetic furrowing

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ABSTRACT During cytokinesis, the cell undergoes a dramatic shape change as it divides into two daughter cells. Cell shape changes in cytokinesis are driven by a cortical ring rich in actin filaments and nonmuscle myosin II. The ring closes via actomyosin contraction coupled with actin depolymerization. Of interest, ring closure and hence the furrow ingression are nonconcentric (asymmetric) within the division plane across Metazoa. This nonconcentricity can occur and persist even without preexisting asymmetric cues, such as spindle placement or cellular adhesions. Cell-autonomous asymmetry is not explained by current models. We combined quantitative high-resolution live-cell microscopy with theoretical modeling to explore the mechanistic basis for asymmetric cytokinesis in the Caenorhabditis elegans zygote, with the goal of uncovering basic principles of ring closure. Our theoretical model suggests that feedback among membrane curvature, cytoskeletal alignment, and contractility is responsible for asymmetric cytokinetic furrowing. It also accurately predicts experimental perturbations of conserved ring proteins. The model further suggests that curvature-mediated filament alignment speeds up furrow closure while promoting energy efficiency. Collectively our work underscores the importance of membrane–cytoskeletal anchoring and suggests conserved molecular mechanisms for this activity.

INTRODUCTION Cytokinesis is the physical division of one cell into two. Cell shape changes during this process are accomplished by a temporary cytoskeletal structure known as the contractile ring, which assembles at the cell equator in anaphase (Schroeder, 1972, 1973; Mabuchi and Okuno, 1977; Rappaport, 1996; reviewed by Green et al., 2012). The contractile ring is a specialization of the cortical actomyosin cytoskeleton, which occupies a thin volume beneath the plasma membrane (Schroeder, 1972; Clark et al., 2013). Key components of this ring include the multidomain scaffold protein anillin, the filament-forming septins, minifilaments of nonmuscle myosin II, and formin-dependent, long, unbranched actin filaments (F-actin). Actin filaments are randomly oriented at the equator at the onset of cytokinesis but become circumferentially aligned (Fishkind and Wang, 1993; Noguchi and Mabuchi, 2001; Vavylonis et al., 2008). Actomyosin contraction coupled to actin depolymerization in the plane of the membrane produces contractile force tangential to the membrane, comprising small radial forces that deform the cell cortex into a furrow that ingresses to divide the cell (Schroeder, 1972).
Furrowing is nonconcentric (asymmetric) in cells throughout Metazoa (Reinsch and Karsenti, 1994; Rappaport, 1996; Das et al., 2003; Alsop and Zhang, 2004; Fleming et al., 2007; Maddox et al., 2007; Kosodo et al., 2008; Carvalho et al., 2009; Bourdages et al., 2014). In most cases, there exists a mechanical asymmetry within the division plane, including intercellular junctions and substrate adhesions, which precedes anaphase. Such features resist furrowing forces and are thus sufficient for asymmetric furrowing (Fournouhou et al., 2013; Guillot and Lecuit, 2013; Hersztorg et al., 2013; Morais-de-Sa and Sunkel, 2013). However, the asymmetry of ring closure can be cell autonomous, as in the Caenorhabditis elegans zygote (Audhya et al., 2005; Maddox et al., 2007). How does this asymmetry arise? Initially, a shallow furrow appears relatively synchronously around the division plane (Maddox et al., 2007). As cytokinesis proceeds, the furrow adopts a sharp leading edge at a singular site (Maddox et al., 2007). This feature is perpetuated such that furrowing occurs unilaterally for some time, and the center of the contractile ring is increasingly offset from the center of the division plane (Maddox et al., 2007). The cortex on the opposite side of the division plane (180° from the initial doubled-membrane feature) adopts this sharp curvature only later.

What is the physical mechanism that leads to asymmetric furrowing? Several theoretical models have been used to study the mechanics of cytokinetic ring contraction (Biron et al., 2005; Wang, 2005; Zumdieck et al., 2007; Koyama et al., 2012; Mendes Pinto et al., 2012; Stachowiak et al., 2014; Turlier et al., 2014), and one describes the mechanical differences around the division plane circumference when asymmetry is imposed (Sain et al., 2015). Still, none explains how cell-autonomous asymmetric closure, such as that in the C. elegans zygote, arises and persists. The prominent accumulation of ring components on the faster-ingressing side of the furrow was suggested to contribute to asymmetry (Maddox et al., 2007). However, diffusion is predicted to smooth out spatial gradients of protein distribution over the several-minute time course of furrow ingression, and thus this factor is likely insufficient to perpetuate asymmetric furrowing. It therefore remains an open question whether asymmetric furrowing causes or results from localized protein accumulation. Another potential player is the anaphase spindle midzone, which comprises bundled antiparallel microtubules between the segregated chromosomes, contributes to division plane specification, and concentrates many positive regulators of cytokinesis (Douglas and Mishima, 2010). Asymmetric placement of the midzone within the division plane could lead to asymmetric contact with the contractile ring and thus stabilize or amplify asymmetry. However, instead of promoting asymmetry, the midzone was implicated in attenuating asymmetry, since asymmetry decreases when the furrow contacts it (Audhya et al., 2005).

We hypothesize that furrowing is nonconcentric via the action of a cortex-intrinsic positive feedback mechanism that perpetuates an initial, stochastic asymmetry. We consider the unique doubled-membrane furrow leading edge as a candidate for the initial asymmetry. By light microscopy, this feature is diffraction limited (Maddox et al., 2005); the radius of membrane curvature is likely <150 nm. Here we propose that high membrane curvature influences the alignment of associated cytoskeletal filaments. That is, for filaments to maximize their association with the membrane and bind without buckling, it is energetically favorable for filaments to align circumferentially along the furrow instead of orthogonally to it. Filament alignment in turn facilitates actomyosin contractility, which further drives furrowing. This concept of membrane curvature-mediated cytoskeletal organization forms the key element in a positive feedback loop among filament alignment, contractility, and membrane curvature.

Here we combine theoretical modeling and experiments to test this idea. Our model predicts that curvature-mediated cytoskeletal organization is essential to recapitulate the observed asymmetric furrow ingression; this prediction is corroborated by experimental perturbations of conserved contractile ring components. The model further suggests functional roles of these proteins and indicates that the asymmetry of furrowing trades off with speed to achieve a maximum energy efficiency of cytokinesis in different cell types. Thus our model encompasses a concept that can explain asymmetric furrowing, describes the mechanics of the contractile ring, and begins to assign molecular identity to these activities. Of importance, we propose a novel contributor to contractile ring mechanics: the influence of local membrane curvature on cytoskeletal organization via filament-membrane attachment.

RESULTS

Model development

Here we present a theoretical model that aims to explain the sustained asymmetry of contractile ring closure in the C. elegans zygote. The model describes the mechanical contraction of the contractile ring and its coupling with the furrowing plasma membrane. The central notion of the model is that membrane curvature in the furrow affects the alignment of the cytoskeletal filaments in the contractile ring, which in turn governs the contractility that further drives furrowing (Figure 1A). We propose that this curvature-mediated feedback loop perpetuates the symmetry breaking that arises from random fluctuations and underlies asymmetric furrow ingression (Figure 1B).

The model describes the cytokinetic furrow ingression without explicit representation of individual molecules. To discern the mechanism underlying asymmetric furrowing, the model comprehensively integrates key components, including the cortical cytoskeleton, the membrane, and the spindle midzone. It builds upon the energy arising from interactions among these key players. We first depict the model qualitatively and then formulate the quantitative model.

Although strict sarcomeric organization has not been observed in cytokinetic contractile rings, several lines of evidence support the idea that the contractile ring comprises a collection of “contractile units” (Bement and Capco, 1991; Carvalho et al., 2009; Pollard, 2010). Motivated by this evidence, we modeled the contractile ring as a circle of connected contractile segments bound to the interior of a membrane tubule, which is 50 μm long with a diameter of 30 μm (Figure 1C), similar to the dimensions of the C. elegans zygote. Within each segment, an ensemble of myosin II bipolar mini-filaments spans two groups of actin filaments (Burns et al., 1995a,b; Yumura et al., 2008; Zhou and Wang, 2008; Vale et al., 2009), which occupy the two circumferential ends of each segment (adjacent to adjoining segments). The external, barbed ends of the actin filaments can be thought of as connected to the neighboring segment by actin cross-linkers, which are represented by the spring-like connections between the segments. Thus the ring segments resemble the sarcomeres of muscle or stress fibers. However, unlike in sarcomeres, the actin filaments within each ring segment begin with random orientation (not aligned) and align progressively with time and as the cytokinetic furrow ingresses (Figure 1C).

The model posits that actin filament alignment within each ring segment arises from two sources. First, we propose that the local membrane curvature favors the circumferential orientation of actin filaments (Figure 1A). This is because when the membrane is flat, filaments can fully attach to the membrane in any orientation (Mukhina et al., 2007; Reichl et al., 2008; Skau et al., 2011), whereas the maximization of filament-membrane binding favors circumferential filament alignment with the furrow when the cortex is curved. We termed this phenomenon curvature-dependent
filament alignment. In addition, F-actin cross-linkers bundle filaments and promote their alignment regardless of the local membrane shape (Mukhina et al., 2007; Reichl et al., 2008; Skau et al., 2011). We further assume that filament alignment facilitates filament sliding and, hence, the contractility of each ring segment. This hypothesis is in part supported by the observation that antiparallel filament orientation potentiates contraction (Reymann et al., 2010). Consequently, contractility coupled with actin depolymerization drives segment shortening and ring closure (Schröder, 1972; Medeiros et al., 2006; Haviv et al., 2008; Carvalho et al., 2009; Wilson et al., 2010; Mendes Pinto et al., 2012). This shortening and the associated inward movement of ring segments further drive the local membrane shape change, thus closing the curvature-mediated positive feedback loop (Figure 1B). On the other hand, aligned, cross-linked F-actin (i.e., an F-actin bundle) is much stiffer than an isotropic meshwork (Gardel et al., 2004; Shin et al., 2004; Claessens et al., 2006). The progressive filament alignment thus also increases the bending resistance of the contractile ring segment, thus opposing the local contractility. Although contractile ring ingression is also opposed by membrane resistance and a resistive force representing the spindle midzone, both of these factors are of relatively low magnitude compared with that of ring rigidity (see model parameter consideration in the Supplemental Material). Thus furrow ingression consists of five tightly coupled subprocesses: filament alignment, sliding, and depolymerization, the displacement of the spindle midzone upon contact with the ring, and membrane shape changes.

Formulation of the quantitative model
We first construct the total energy function of the contractile apparatus $F$, which sums the mechanical energy of the system and free energy changes arising from the ordering of the F-actin within each ring segment:

$$F = F_{\text{membrane mechanics}} + F_{\text{ring mechanics}}$$

The mechanical energy includes membrane mechanics, the mechanics of the contractile ring, and the interaction between the contractile ring and the midzone spindle. We consider that filament alignment within the contractile ring is a phase transition process. The associated free energy change is

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**FIGURE 1:** Schematics of a theoretical model featuring membrane curvature-mediated feedback between filament alignment and contraction in asymmetric furrowing. (A) A key model concept is curvature-dependent filament alignment. Membrane curvature favors filament alignment along the furrow. (B) Positive feedback loop among local filament alignment, contractile force, and membrane curvature. (C) Model layout and coordinate system. (D) Magnified view of sarcomere-like ring segment described in the model and considerations of force generation within segments and contraction effects on neighboring segments. (E) Schematics of order parameter for filament alignment.
modeled as a continuous phase transition and described by the “ψ–4 theory,” which is used to characterize phase transitions in soft condensed matter such as liquid crystals (Landau and Lifshitz, 1980). We construct our model in a discrete form in order to faithfully represent actomyosin contractility (see Section I in the Supplemental Material for detailed reasons and Table 1 for term definitions). Later we will elaborate on each energy contribution.

\[ F_{\text{Membrane mechanics}} = \sum_{z=-N_z}^{N_z} \sum_{s=1}^{N_s} g_3(s, z, t) \left[ \frac{\kappa}{\rho} C_0 g_z^2(s, z, t) + \frac{1}{2} \left( x_{s} y_{s} - x_{d} y_{d} \right) \right] \]  

Equation 2 describes the membrane mechanical energy, which is Helfrich-like membrane energy in its continuum limit, and includes contributions of bending energy, surface tension, and osmotic pressure. Throughout the model, a cylindrical coordinate system is used. The membrane tubule is divided into \( N_z + 1 \) parts along the z-axis (the pole-to-pole axis) and \( N_s \) segments in the radial direction, where \( s \) stands for the \( s \)th membrane patch in the radial direction (Figure 1C). Under this coordinate system, the position of the membrane patch \( r(s, z, t) = (x(s, z, t), y(s, z, t)) \) is represented as a function of \( s, z, \) and \( t \). From the differential geometry, the mean curvature of the local membrane is

\[ C_M = \frac{1}{2 g_z^2} \left[ (x_s y_s - x_d y_d) + x_s^2 - y_s^2 + y_s^2 - x_s y_s - 2 (x_s x_d + y_s y_d) (x_s y_d - x_d y_s) \right] \]

Here \( g_3(s, z, t) = x_s^2 + y_s^2 + (x_s y_d - x_d y_s)^2 \). Finally, all of the expressions of derivatives are in discrete form; for example, the partial derivative of \( x \) with regard to \( s \) is \( x_s = \frac{1}{2} (x(s+1) - x(s-1)) \).

We have

\[ F_{\text{Ring mechanics}} = \sum_{s=1}^{N_s} \left[ \left( \frac{1}{2} \left( f_r(s, t) f_r(s, t) - f_r(s+1, t) f_r(s-1, t) - f_r(s, t) f_r(s+1, t) f_r(s-1, t) \right) \right) \right] \]  

Equation 3 characterizes the contractile ring mechanics, including the contraction, connectivity, bending, and ring–midzone spindle interaction. Here \( f_r(s, t) \) and \( f_l(s, t) \) are the internal contractile forces on the left and right ends for the \( s \)th ring segment, respectively (Figure 1D). In addition, \( f_r(s, t) + f_l(s, t) = 0 \) and \( f_l(s) = f_l(t) \) is the unit of force per length and \( f_r(s, t) = f_r(t) (s, t) = f(s), \) where \( f(s) \) has the unit of force per length and \( f_l(s, t) = l..(s, t) = \frac{1}{2} (l..(s, t), \) where \( l.., t \) is the length of the actin filaments overlapping with the myosin II heads within the ring segment. Because the ring is constitutively attached to the membrane at the equator \( z = 0 \), the model dictates that the position of the \( s \)th ring segment is the same as that of the corresponding membrane patch. The position of the ring segment is similarly represented as

\[ r(s, z, t) = (x(s, z, t), y(s, z, t)) \]

which is reduced to \( r(s, t) = (x(s, t), y(s, t)) \) for simplicity. Here \( g_2(s, z, t) = x_s^2 + y_s^2 \) and \( \sqrt{g_2(s, t)} \) is the end-to-end length of the \( s \)th ring segment (Figure 1D). In addition, because actin cross-linkers connect the ring segments, force balance dictates that this mismatch within the ring segment could stretch or compress these cross-linkers between the neighboring ring segments (see Sections II and III in the Supplemental Material for detailed parameter derivation together with references). A mismatch between the myosin II filaments length \( \sqrt{g_2(s, t)} \) and actin filament length \( l(s, t) \) (Figure 1D) could thus incur an energy penalty, approximated by \( \frac{1}{2} K_1 (\sqrt{g_2(s, t)} - l(s, t))^2 \) in the model. The local curvature of the ring is \( \kappa r_m = (x_s y_d - x_d y_s) / g_2^{1.5} \), and the position of the spindle midzone is denoted as \( \kappa r_m \).

\[ F_{\text{Free energy from filament alignment}} = \sum_{s=1}^{N_s} \left\{ \psi(s, t) + (a_0 + AC_0) \psi^2(s, t) + B \psi^4(s, t) \right\} \]

Equation 4 represents the free energy change from filament ordering. The order parameter \( \psi(s) \) is a normalized value that describes the level of F-actin alignment ordering within each ring segment. We provide a detailed physical derivation of \( \psi(s) \) in Section I in the Supplemental Material. Briefly, \( \psi > 0 \) when filaments are antiparallel with pointed ends facing toward each other, \( \psi < 0 \) for those in parallel, and \( \psi = 0 \) when filaments are in total disorder (Figure 1E). Of importance, two terms in Eq. 4 drive the filament alignment order parameter \( \psi \). First, the term \( a_0 \) represents the energetically favorable filament bundling driven by F-actin cross-linkers; \( a_0 \) is a negative constant, driving filament alignment regardless of local membrane shape. Second, the term \( AC_0 \) specifically drives filament alignment via local Gaussian curvature \( C_G \) (Figure 1A), where

\[ C_G = g_3^{-2} \left[ (x_s y_d - x_d y_s) (x_s y_d - x_d y_s) - (x_s y_d - x_d y_s)^2 \right] \]

A represents the effective filament–membrane binding energy, which is positive (see Sections I and III in the Supplemental Material for detailed derivation). At the local furrow (negative local Gaussian curvature), the term \( AC_0 \) turns negative, and thus curvature-dependent filament alignment is favorable (Figure 1A). In addition, the term \( D \) in the phase ordering free energy represents the energy penalty for spatial variation in \( \psi \), thus promoting the uniform ordering in filament alignment along the ring perimeter. The fourth-order term is positive and an entropy term that favors disorder in filament alignment.
Critically, the local filament ordering affects the contractility $f(s)$ and the stiffness of the ring segment $\kappa_3$. Specifically,

$$f(s) = f_0 \cdot \left(\psi(s) + |\psi(s)| \right) \left(1 - \frac{\alpha}{R_0} |f(s) - \bar{r}_m|\right)$$

and $\kappa_3 = \kappa_3^0 |\psi|$. Here $f(s)$ is assumed to be proportional to $(\psi + |\psi|)$: only the organization in which filaments are antiparallel with pointed ends facing toward each other ($\psi > 0$) can confer productive myosin II contraction; $f_0$ is the maximum contractile force per unit length; and $R_0$ is the original radius of the ring. The $\alpha$ term
Because the shrinkage of actin filament length ($\lambda$) is an active process, it is assumed to be directly driven by the local contraction (see Section I in the Supplemental Material for detailed theoretical considerations). The $\lambda_i$ ($i = 1–4$) in the following equations are the effective viscous drag coefficients for ring contraction and membrane shape change ($\lambda_1$), filament shrinkage ($\lambda_2$), filament alignment ($\lambda_3$), and midzone spindle displacement ($\lambda_4$), respectively:

$$\lambda_1 \frac{\partial r}{\partial t} = -\frac{\delta F}{\delta (s, z, t)}$$

$$\lambda_2 \frac{\partial \psi}{\partial t} = -f(\psi(s, t)) \sqrt{g_2(s, t)}$$

$$\lambda_3 \frac{\partial \psi}{\partial t} = -\frac{\delta F}{\delta \psi(s, t)} + \zeta(s, t)$$

$$\lambda_4 \frac{\partial \tilde{m}}{\partial t} = -\frac{\delta F}{\delta \tilde{m}(s, z, t)}$$

To calculate furrowing dynamics, we numerically integrate Eqs. 5–8 over time from the initial state in which filament orientation is random and contraction force is zero, the midzone is centered on the origin, and the membrane tubule is at mechanical equilibrium. We consolidated all of the randomness of the system into the phase ordering process in Eq. 7 by a white Gaussian noise $\langle \zeta(s, t) \rangle = 0$ and $\langle \zeta(s_1, t_1) \zeta(s_2, t_2) \rangle = \sigma_0^2 \Delta(s_1 - s_2) \delta(t_1 - t_2)$. At each time step, the model reports the state of the system by the position of each membrane patch and midzone spindle and the position, the length, and the filament ordering level of each ring segment.

The key model result of asymmetric furrowing is robust against variations of the model parameters (Supplemental Figure S1). It emerges as a natural consequence of the curvature-dependent filament alignment, largely independent of specific model parameter values.

The theoretical model recapitulates asymmetric furrowing of the C. elegans zygote when contraction is faster than filament alignment.

We first tested whether our model based on a simple mechanical feedback loop can recapitate experimentally observed kinetics and geometry of cytokinesis. We performed high-resolution three-dimensional time-lapse imaging of C. elegans zygotes and measured contractile ring size and position over time (Figure 2, A–C, and Supplemental Movie S1; Dorn et al., 2010; Bourdages et al., 2014).

Our model was able to recapitate three aspects of the kinetics of cytokinesis in vivo (Figure 2, C and D): initiation timing, closure...
duration, and nonconcentricity. According to our model, actin filaments are initially randomly oriented ($\psi = 0$ everywhere). The model assumes that a small and constant driving force ($\alpha_0 < 0$ in Eq. 4) promotes circumferential filament alignment. Thermal fluctuations, however, seed the initial heterogeneity in $\psi$ along the perimeter (Figure 2Ei). As more filaments become locally aligned, actomyosin contraction becomes more efficient (Figure 2Eii). The resulting stronger contraction drives a deeper furrow formation, the larger Gaussian curvature of which locally promotes a greater filament alignment that in turn facilitates even more local contractions (Figure 2Eiii). Thus the local curvature mediates positive feedback between the cytoskeletal organization that facilitates force generation and the resulting furrow ingress, amplifying asymmetry.

For the initial asymmetric furrow to be perpetuated, our model predicts that the pace of filament sliding (and thus ring contraction) is faster than the filament alignment pace. This is because when the furrow starts to ingress locally, the membrane curvature of the adjacent region will gradually increase, in turn locally promoting filament alignment. Thus filament alignment, contraction, and furrowing will spread circumferentially from the initial furrowing site to the entire ring (Figure 2Eiv). Only when the rate of filament alignment is much slower than that of ring contraction can the initial asymmetry be amplified and sustained far into furrow ingress (Figure 2F). We thus found that four of the model parameters were critically important in dictating the dynamics of ring contraction: the free energy gains from filament bundling and for curvature-dependent filament alignment and the rates of filament alignment and sliding (Figure 2G). More specifically, whereas the free energy gains for filament bundling and curvature-dependent filament alignment ($\alpha_0$ and $\alpha$ in Eq. 4) promote ring closure, closure dynamics is slowed by the associated viscous drag coefficients of the dynamic processes ($\lambda_1$ in Eq. 5 and $\lambda_2$ in Eq. 7).

**The nature of contractile ring actin influences furrow asymmetry**

The model predicts that when contraction is slow relative to filament alignment, the initial ingestion propagates more quickly around the cell circumference and eliminates the initial asymmetry (Figure 2F). To test this model prediction in vivo, we depleted the protein ARX-2, a member of the well-characterized, conserved Arp2/3 complex, which regulates the actin cytoskeleton in many contexts (Firat-Karalar and Welch, 2011) and was implicated in negative regulation of contractility during cytokinesis (Moulding et al., 2007; Canman et al., 2008) but whose depletion or mutation does not grossly alter contractility (driven by filament alignment). For the initial asymmetric furrow to be perpetuated, our model result demonstrates the necessity of curvature-mediated filament alignment and sliding (Figure 2F; see later discussion). As predicted for decreased cytoskeletal–membrane coupling, curvature-dependent filament alignment was made less energetically favorable (smaller $\alpha$) and the filament alignment pace was increased (smaller $\lambda_2$), since anchoring is needed to translate contractility into furrow ingress (Figure 3, B and D). By decreasing the energy incentive for curvature-mediated filament alignment, the reduction in membrane–cytoskeleton attachment compromises curvatures-mediated positive feedback. Consequently the membrane curvature resulting from furrowing does not influence filament alignment locally. As a result, curvature-independent filament alignment dominates, circumferential filament alignment is more uniform, and furrowing is more symmetric (Figure 3, B and C, solid lines).

Indeed, depletion of anillin or the septins results in concentric ring closure (Figure 3, B and C; Maddox et al., 2007). We fitted the model to three aspects of experimental ring closure dynamics and geometry: initiation timing, duration of closure, and asymmetry. Fitting the model to measurements from cells depleted of anillin or of the septins required varying the four critical model parameters (Figure 3D; see later discussion). As predicted for decreased cytoskeleton–membrane coupling, curvature-dependent filament alignment was made less energetically favorable (smaller $\alpha$) and the filament alignment pace was increased (smaller $\lambda_2$). The necessary decrease in the filament sliding rate (larger $\lambda_1$) could be conferred by a reduction in cytoskeleton–membrane attachment. This model result demonstrates the necessity of curvature-mediated positive feedback for asymmetric furrow ingress. In addition, these results support the idea that anillin and the septins contribute to cytokinesis by coupling actomyosin to the membrane (Pieknny and Maddox, 2010; Saarikangas and Barral, 2011; Liu et al., 2012).
Feedback drives cytokinesis asymmetry

Of interest, anillin and septin depletions have opposite effects on furrow initiation timing: septin depletion allows earlier initiation, whereas anillin depletion causes a delay (Maddox et al., 2007; Figure 3, B and C). To recapitulate the kinetics as well as the asymmetry results of the two perturbations, we made opposite alterations of one model parameter. To fit anillin depletion ring closure kinetics, we made filament bundling less energetically favorable ($a_0$ in Eq. 4 is less negative; Figure 3D), indicating that anillin’s F-actin-bundling activity (Field and Alberts, 1995; Maddox et al., 2005; Dorn et al., 2010) is important for its contributions to ring closure dynamics. Indeed, anillins bundle F-actin in vitro (Supplemental Figure S2A; Field and Alberts, 1995; Kinoshita et al., 2002). In contrast, an increase in free energy gain from filament bundling ($a_0$ in Eq. 4 is more negative) is necessary to recapitulate the earlier initiation resulting from septin depletion (Figure 3D). The reduction of membrane binding–based friction, overabundance of myosin II (Supplemental Figure S2B; Maddox et al., 2007), and lack of effect on anillin recruitment (Supplemental Figure S2C) likely facilitate bundling when septins are depleted.

Myosin II is believed to drive contractile ring closure by actomyosin filament sliding or processively cross-linking depolymerizing actin filaments (Green et al., 2012; Ma et al., 2012; Mendes Pinto et al., 2012). Consequently myosin II inhibition could be expected to reduce the rate of contraction (i.e., larger viscous drag coefficient $\lambda_1$ or, equivalently, smaller contraction force $f_0$) and thus cause slower, more-symmetric furrowing (Figure 3E). Instead, although partial myosin II inhibition by depletion of the myosin light chain activator Rho kinase predictably delays furrow initiation and slows ingression, it in fact increases furrow asymmetry (Piekny and Mains, 2002; Maddox et al., 2007). In addition, depletion of NOP-1, a condition also believed to decrease activity of Rho effectors, results in markedly asymmetric furrowing (Tse et al., 2012). Our model suggests that increased asymmetry can result from an exaggerated increase in the rate of contraction relative to that of filament alignment (smaller
We thus hypothesize that if myosin II contributes not only to contractile force, but also to filament alignment, the contractile force, are set to zero. For model result in D, in addition to the model parameter changes in C, other parameter alterations are similar to that in Figure 3, B and D, to mimic the effect of anillin depletion.

\( \frac{\lambda_1}{\lambda_3}; \) Figure 3, D and E). One way in which myosin II inhibition could cause this imbalance is if myosin II contributes not only to contraction, but also to filament alignment. In support of this idea, myosin II exists in the contractile ring as bipolar filaments (Yumura et al., 2008; Zhou and Wang, 2008; Vale et al., 2009; Beach et al., 2014) that robustly bundle actin in vitro (Reymann et al., 2010; Thoresen et al., 2011). In addition, myosin II, like other actin bundlers, could slow filament sliding and hence contraction by generating friction (Janson et al., 1992; Mukhina et al., 2007). Partial inhibition of myosin II would thus be predicted to reduce friction and impede filament alignment and therefore increase the contraction speed relative to that of filament alignment. In sum, due to the increased asymmetry seen after myosin II inhibition, our model suggests dual roles for myosin II in cytokinesis: driving contractility and bundling actin.

**The geometry of furrowing is largely intrinsic to the cortex**

The spindle midzone comprises antiparallel bundled microtubules and lies in the center of the division plane. A nonconcentric furrow encounters the midzone when it is approximately half closed (Figure 4A). On impact, the midzone may attenuate the asymmetry of the furrow by physically resisting its ingression or it may boost asymmetry because it harbors activators of contractility. Our model predicts that the asymmetry of furrow ingression is largely intrinsic to the cell cortex since the forces exerted by the contractile ring are much larger than those holding the midzone in place (Supplemental Material). In silico, the ring closes asymmetrically even in the absence of resistive forces representing the spindle midzone (Figure 4B, C, and D). Indeed, the midzone is readily displaced upon impact with the furrow in control cells (Figure 4A). To test whether the midzone attenuates asymmetry, we depleted the microtubule bundler SPD-1, eliminating midzone microtubule bundles but not blocking cytokinesis completion (Verbrugghe and White, 2004; Figure 4B and Supplemental Figure S3, A and B). Furrow asymmetry is slightly higher in SPD-1-depleted cells than in controls (Figure 4, B and C, and Supplemental Figure S3A), suggesting that the midzone is a weak barrier to furrow ingression. Therefore the midzone is not necessary for asymmetry.

The furrow can initiate asymmetrically but terminate close to the center of the division plane (it can recenter) after perturbation of cortical proteins (Figure 3). Although our model suggests that asymmetry is intrinsic to the cortex, recentering could result from the balance of forces between the cortex and midzone. We directly tested whether recentering requires resistance by the midzone or can occur in the absence of midzone bundles. We combined thorough depletion of SPD-1 with a cortical perturbation that elicits recentering (partial depletion of anillin; Supplemental Figure S3, C and D). Alteration of the cortical cytoskeleton was thus sufficient to dictate the geometry of ring closure (recentering), even in the absence of a midzone barrier (Figure 4A). Therefore we conclude that ring geometry is primarily dictated by cortical players.

**Asymmetric furrowing increases energy efficiency**

We previously showed that reduction of active myosin II causes failure of symmetric cytokinesis, even though asymmetric furrowing can succeed in this condition (Maddox et al., 2007). We thus...
hypothesized that symmetric furrowing requires more energy than asymmetric closure. We tested this hypothesis computationally by integrating the total energy expenditure for contractile ring closure in scenarios of different relative rates of filament sliding and alignment while keeping other model factors unchanged. That is, we numerically calculated the work done during furrowing by integrating the driving force of filament contraction (the right-hand side of Eq. 1) times the displacements of the ring over time. We found that asymmetric furrow closure is ~50% more energy efficient than symmetric furrow closure (Figure 5A). Although filament alignment facilitates contraction, it also increases the rigidity of the cortical cytoskeleton and consequently the bending resistance within the contractile ring. In a symmetric contractile ring, where alignment is uniformly high, ingestion thus costs more energy. This suggests that the relationship between the paces of filament alignment and of sliding, which dictates the asymmetry of furrow ingression, determines the energy efficiency of cytokinesis.

**Energy efficiency of furrow ingression trades off with speed**

Energy efficiency is unlikely to be the only factor that influences cell behavior. In embryos, speed is of the essence, cell cycles are short, and cell division events are fast, as compared with their counterparts in somatic cells. We leveraged our model to test whether removing the constraint of closure speed could reveal a more energy-efficient mode. We reduced curvature-dependent filament alignment (i.e., decreasing the value of \( \alpha \) in Eq. 4 while keeping all of the other parameters the same) such that curvature-mediated positive feedback is eliminated (while keeping all other factors unchanged). With decreased \( \alpha \), the energy efficiency of ingression increases (Figure 5B, red trace). Decreasing the curvature-dependent filament alignment factor \( \alpha \) by 100 times from that for the control causes ring closure to take more than six times longer than in the C. elegans zygote (Figure 5B, blue trace), longer than the entire C. elegans embryonic cell cycle (Deppe et al., 1978) but fully compatible with the kinetics of cytokinesis in somatic cells. In this mode, ring closure is relatively symmetric (Figure 5A). In cells, this likely corresponds to a broad contractile apparatus such as that of mammalian and *Drosophila* cultured cells, which yields a very small Gaussian curvature at the furrowing site. The model further reveals that, although the energy gain from curvature-dependent filament alignment greatly increases furrowing speed, it also increases energy cost (Figure 5B, red trace). The model thus suggests that the strength of curvature-dependent filament alignment dictates different modes of cytokinetic furrow ingression in different cell types, where speed trades off with energy efficiency.

**DISCUSSION**

A novel model for cytokinesis implementing membrane curvature-dependent cytoskeletal organization explains asymmetric furrowing

Here we investigate the physical mechanism of cytokinetic ring closure. Our model based on mechanical feedback among filament alignment, contraction, and membrane curvature recapitulates not only the kinetics of cytokinesis but also its asymmetry, a phenomenon that is ubiquitous but unexplained in its cell-autonomous instances. Central to the feedback is curvature-dependent filament alignment (Figure 1A). Furthermore, protein-based membrane–cytoskeleton linkers (e.g., anillin and the septins) are essential for this feedback, as they govern the relationship between the paces of contraction and filament alignment and consequently the balance between furrow ingression and furrow propagation. This balance, rather than differences in overall mechanism, likely accounts for the observed differences in the geometry of ring closure among cell types.

Of interest, while not consistently observed, a subset of controls contains cells that display spiraling movement of the cytokinetic ring center in addition to asymmetric furrowing (Supplemental Figure S4). To date, our theoretical model has not been able to recapitulate this large-scale spiral movement of the ring center. This is because the model describes membrane as an elastic sheet and imposes that the ring segment and the membrane patch to which it attaches share the same spatial position. The observed spiral movement reflects the ring rotation, which in reality likely entails dissociation between the ring segment and the local membrane and/or rearrangement of the associated membrane patches in the plane of the membrane. The present model cannot describe either of these two events sufficient for ring rotation. In particular, the in-plane rearrangement of membrane patches requires the lateral movement of lipids in the membrane plane. It is hydrodynamic in nature, which cannot be faithfully described by the elastic illustration in the model, which assumes that membrane patches do not rearrange their relative position. This intriguing spiraling phenomenon thus warrants future investigation.

The model allows functional annotation of conserved contractile ring proteins

The simplicity of our model and absence of protein-based agents allow an unbiased investigation of protein function. Because we found that the mechanics of cytokinesis is largely intrinsic to the cortical cytoskeleton, we focused on annotating several conserved ring components—anillin, septins, and myosin II—which have been proposed to have multiple, overlapping functions. Fitting the model to the kinetics of three independent experimental readouts (initiation timing, closure duration, and asymmetry) upon protein perturbations allows us to make specific mechanistic hypotheses about the roles of these ring components in membrane–cytoskeleton coupling, filament bundling, and contractility.

Anillin, a large multidomain protein that binds many cytokinetic ring components, has engendered myriad hypotheses for its mode of action (Piekny and Maddox, 2010). Our model suggests that
anillin contributes to actomyosin filament alignment, likely via its F-actin-bundling domain but also possibly by linking F-actin with active myosin II (Field and Alberts, 1995; Straight et al., 2005). Model fitting also suggests that anillin contributes to cytoskeleton-membrane linkage. Indeed, anillin bears a lipid-binding PH domain (Liu et al., 2012) and can bind the septins (Kinoshita et al., 2002). An important role in cytoskeleton-membrane linkage may explain the division plane instability in anillin-depleted cultured mammalian and Drosophila cells (Piekný and Maddox, 2010). Although these roles are not unexpected, our work suggests that this particular behavior is a major factor in how anillin contributes to contractile ring function.

Septins’ roles in metazoan cytokinesis are poorly defined and hypothesized based on their in vitro activities and roles elsewhere. As linear and grid-like polymers, they could aid cytoskeleton alignment. As membrane associated proteins, they could link the cytoskeleton to the membrane. In most organisms, many septin proteins and isoforms are expressed and likely form multiple combinatorial complexes (Sellin et al., 2011). In C. elegans, a single heterotetrameric species is believed to be formed from the only two septin proteins (John et al., 2007), and septin function can be completely removed with a single molecular perturbation (Nguyen et al., 2000). Our model suggests that septins are important for membrane-cytoskeletal linkage. This proposition is consistent with the role of septins in bleb retraction (Gilden et al., 2012).

Our model suggests that myosin II not only provides contractility, but it also contributes significantly to actin cross-linking and bundling. This result likely relates to the fact that the myosin II in contractile ring is in the form of bipolar miniﬁlaments (Yumura et al., 2008; Beach et al., 2014). Myosin II’s cross-linking activity has been implicated in providing the driving force for cytokinetic furrowing (Yumura et al., 2008; Mendes Pinto et al., 2012; Beach et al., 2014). Going forward, it will be important to uncover the relative contributions of myosin II’s multiple activities.

Together these results demonstrate the power of our model in annotating the functions of conserved contractile components.

Further considerations of curvature-mediated positive feedback

The key factor of our model is that the curvature of the furrow is both the cause and the result of contractile ring constriction and furrow ingestion, that is, curvature-mediated positive feedback. With a minimal model describing the essential elements of the system, we recapitulated furrow asymmetry with a positive feedback loop mediated by curvature-dependent filament alignment. This feedback concept is based on the energy consideration that actin filaments maximize their attachment to the membrane and free energy is reduced when filaments orient circumferentially along the furrow (Figure 1A). The model assumes that the actin filaments are straight throughout their length but could buckle with a 300-nm radius of curvature (Murrell and Garde, 2012). Consequently they could partially conform to the high curvature at the furrowing site (likely <150-nm radius of curvature) and align transversely across the furrow. Nevertheless, filament buckling is energetically unfavorable. Thus our proposed configuration of straight actin filaments with circumferential alignment has a relatively lower free energy and is more favorable. Future work will be aimed at precisely measuring furrow curvature and its relationship to local cytoskeletal orientation.

Other factors could contribute to the curvature-mediated positive feedback. For instance, curvature-stabilizing proteins may respond to and further stabilize the membrane curvature at the furrowing site. These include proteins containing F-BAR domains (Shlomovitz and Gov, 2008) or amphipathic helices or that form curved polymers, such as septins (Kinoshita et al., 2002; Tanaka-Takiguchi et al., 2009). These factors could thus take part in driving cytokinesis by promoting the positive feedback loop we propose here.

Myosin II has been shown to contract more powerfully in response to mechanical load (Cremo and Geeves, 1998; Veigel et al., 2003; Kovas et al., 2007). Increased filament alignment due to bundling or curvature-dependent filament alignment would allow engagement of more myosin II heads on actin and thus more load. This could in turn lead to more powerful myosin II contraction, potentially myosin II filament growth (Ren et al., 2009), and deeper ingress. Therefore load-sensitive contractility would participate in the same feedback loop we propose.

One model to explain different modes of cytokinesis

Asymmetric furrowing occurs in most, but not all, popular model species. Most prominently, budding and fission yeast exhibit symmetric ring closure (Bi et al., 1998; Wu et al., 2003). This behavior is predicted by our model: within the parameter ranges tested, when the diameter of the cell is <5 μm, circumferential propagation of the initial asymmetric furrow completes so quickly that furrowing becomes symmetric (Supplemental Figure S1C). In addition, circumferential furrow propagation may be favored by cytoskeletal arrangement; in budding and fission yeasts, the septin and actomyosin cytoskeletons, respectively, become circumferentially aligned before the onset of ring closure (Vrabioiu and Mitchison, 2006; Vavylonis et al., 2008). According to our model, this phenomenon corresponds to filament bundling outpacing filament sliding, which results in symmetric furrowing.

Asymmetric furrowing is an energy-efficient way to perform fast cytokinesis, according to our model. Speed is very important in embryonic systems. Indeed, many species’ embryos exhibit asymmetric furrowing. Whereas in the C. elegans zygote the mechanical feedback loop may be sufficient to initiate asymmetric furrowing, in other systems, such as the ctenophore zygote, supplementary mechanisms including eccentric spindle placement account for asymmetry. For epithelial cells in a monolayer, mechanical resistance by apical junctional complexes biases the effectiveness of equatorial contractility to the basolateral cell surface (reviewed by Bourdages and Maddox, 2013). Similarly, remnant substrate adhesions likely locally attenuate furrowing upward from the coverslip during asymmetric furrowing in cultured human and Drosophila cells (Bourdages et al., 2014). In all of these cases, the feedback loop we propose likely contributes to the maintenance of asymmetry. In future work, it will be interesting to examine furrowing kinetics and geometry in somatic cultured cells in mechanical isolation. Given the conservation of the structural proteins implicated in our positive feedback loop, we expect that in such cells, furrowing will be measurably asymmetric even in the absence of extrinsic cues.

MATERIALS AND METHODS

See the Supplemental Experimental Procedures for further considerations of the theoretical model.

C. elegans maintenance and sample preparation

C. elegans were maintained according to standard procedures at 20°C. Strains used were JJ1473 (Munro et al., 2004; Carvalho et al., 2009) and OD122 (Dorn et al., 2010). See Supplemental Table S1 for full genotypes. Embryos were dissected from gravid hermaphrodites and mounted under coverslips on agarose cushions as in Gonczy et al. (1999).
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