Deep elastic strain engineering of bandgap through machine learning

Zhe Shi¹b,¹, Evgenii Tsymbalov¹c,², Ming Dao*, Subra Suresh¹b,², Alexander Shapheev²,², and Ju Li¹b,²

¹Department of Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139; ²Department of Nuclear Science Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139; ³Skolkovo Institute of Science and Technology, 121205 Moscow, Russia; and ⁴Nanyang Technological University, 639798 Singapore, Republic of Singapore

Contributed by Subra Suresh, December 18, 2018 (sent for review November 1, 2018; reviewed by Yonggang Huang and Devendra K. Sadana)

Nanoscale specimens of semiconductor materials as diverse as silicon and diamond are now known to be deformable to large elastic strains without inelastic relaxation. These discoveries harbinger a new age of deep elastic strain engineering of the band structure and device performance of electronic materials. Many possibilities remain to be investigated as to what pure silicon can do as the most versatile electronic material and what an ultrawide bandgap material such as diamond, with many appealing functional figures of merit, can offer after overcoming its present commercial immaturity. Deep elastic strain engineering explores full six-dimensional space of admissible nonlinear elastic strain and its effects on physical properties. Here we present a general method that combines machine learning and ab initio calculations to guide strain engineering whereby material properties and performance could be designed. This method invokes recent advances in the field of artificial intelligence by utilizing a limited amount of ab initio data for the training of a surrogate model, predicting electronic bandgap within an accuracy of 8 meV. Our model is capable of discovering the indirect-to-direct bandgap transition and semiconductor-to-metal transition in silicon by scanning the entire strain space. It is also able to identify the most energy-efficient strain pathways that would transform diamond from an ultrawide-bandgap material to a smaller-bandgap semiconductor. A broad framework is presented to tailor any target figure of merit by recourse to deep elastic strain engineering and machine learning for a variety of applications in microelectronics, optoelectronics, photonics, and energy technologies.

Significance

Deforming a material to a large extent without inelastic relaxation can result in unprecedented properties. However, the optimal deformation state is buried within the vast continua of choices available in the strain space. Here we advance a unique and powerful strategy to circumvent conventional trial-and-error methods, and adopt artificial intelligence techniques for rationally designing the most energy-efficient pathway to achieve a desirable material property such as the electronic bandgap. The broad framework for tailoring any target figure of merit, for any material using machine learning, opens up opportunities to adapt elastic strain engineering of properties and performance in devices and systems in a controllable and efficient manner, for potential applications in microelectronics, optoelectronics, photonics, and energy technologies.


Reviewers: Y.H., Northwestern University; and D.K.S., IBM T. J. Watson Research Center.

Conflict of interest statement: The authors have declared no conflict of interest presented in this paper.

This open access article is distributed under the Creative Commons Attribution-NonCommercial-NoDerivatives License 4.0 (CC BY-NC-ND).

¹Z.S. and E.T. contributed equally to this work.
²To whom correspondence may be addressed. Email: ssuresh@mit.edu, A.Shapheev@skoltech.ru, or liju@mit.edu.

This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1818555116/-/DCSupplemental.

Published online February 15, 2019.
information, and energy applications that impact every aspect of modern life (9).

ESE seeks to identify metastable states of matter for optimizing functional properties and performance. A strained material is in a state of higher energy than when it is in a stress-free state, characterized by the strain-energy density $h$ which is measured in units of meVÅ$^2$. Therefore, addressing the following question is at the heart of ESE: What is the energy cost ($h$) to achieve the desired property change? Consider the challenges of reducing the bandgap of Si from 1.1 eV in its stress-free state to 0 eV in a metal-like state, or converting diamond from an ultrawide-bandgap material into a wide or even medium-bandgap material so that the full potential of its many appealing characteristics for microelectronics and optoelectronics could be realized. To achieve the above transitions in the most efficient manner, it is important to design $\varepsilon$ through the most optimal combination of its normal and shear components.

To address the foregoing question, we resort to deep ESE which exploits the latest advances in artificial intelligence and multiscale modeling. To set the scene, consider a situation where it is desirable to examine all possible combinations of the components of $\varepsilon$, over a range of potential interest, say between $-10$ and $+10\%$ in each strain component. Here, say that the objective is to determine the least energetically expensive route to alter the bandgap of a material by a desired amount. Although ab initio calculations such as those involving many-body corrections can provide accurate energy-band results, the scope of such calculations is somewhat limited to about 1,000 strain points because of high computational cost. On the other hand, by discretizing $\varepsilon$ with a regular grid comprising 20 nodes separated at each 1% strain interval over the strain range of $-10$ to $+10\%$, the computational model would entail about $10^6$ band structures, up to five orders of magnitude higher computational requirement than what can be reasonably achieved presently. To overcome these difficulties, we present here a general method that generalizes machine learning (ML) and ab initio calculations to identify pathways to ESE. This method invokes artificial neural networks (NNs) to predict, to a reasonable degree of accuracy, material properties as functions of the various input strain combinations on the basis of only a limited amount of data. We also demonstrate the potential of our method for bandgap engineering with specific calculations for perfect crystals of Si and diamond. These two materials bookend the wide spectrum of current possibilities and potential opportunities for optimizing the performance of semiconductor materials and devices. Si, on the one hand, represents the most widely used and commercially successful semiconductor material. Diamond, on the other hand, represents the most appealing ultrawide-bandgap material due to its extremely high thermal conductivity and hardness, high electron/hole mobilities and saturation drift velocities, and breakdown field (10). Tuning bandgap, and more broadly the band structure, through deep ESE provides opportunities for tapping into the many appealing figures of merit for device performance of any material. Moreover, we choose Si, the most versatile electronic material, to demonstrate that our ML machinery is capable of predicting important physical phenomena such as indirect-to-direct bandgap transition and semiconductor-to-semimetal transition. We also visualize silicon’s “paleolith”-like isolandgap surfaces in strain space, akin to the yield surface commonly used to describe the plasmonic deformation of metallic materials, but with sharp ridges and corners that reflect band-edge crossovers.

Results
ML and Density of States of Bandgap. We aim to describe the electronic bandgap and band structure as functions of strain by training ML models on first-principles density-functional-theory (DFT) data. This approach leads to reasonably accurate training with much fewer computed data than fine-grid ab initio calculations and a fast evaluation time. The DFT calculations were conducted in two settings: a large, computationally inexpensive Perdew–Burke–Ernzerhof (10) (PBE) dataset obtained for fitting and a small but accurate many-body GW [G, Green’s function; W, screened Coulomb interaction (11)] dataset for correction. As depicted in Fig. 1A, the strain tensor and/or the k-point coordinates are fed into different ML models as input to fit or make predictions about energy eigenvalues or bandgap. Table 1 demonstrates the accuracy of these models on the PBE data, the best of which is attained by the NN. The data fusion technique (12, 13) is adopted to further improve the learning outcome of bandgap. The resulting model allows the prediction of bandgap to reach an extremely high accuracy of 8 meV in the mean absolute error (MAE), as shown in Fig. 1B and SI Appendix, Table S1. The successful combination of the quantitative advantage of PBE and the qualitative advantage of GW results in a bandgap-prediction model with a level of accuracy comparable to experiments.

Fig. 1. (A) ML workflow with NN. For a typical bandgap-prediction task, the input contains the strain information only and the target is either $E_{\text{PBE}}$ or $E_{\text{GW}}$. In the data fusion process, the bandgap predicted from fitting the PBE dataset is also taken in as an input to fit the GW bandgap. For the whole band structure fitting task, the input contains both strain information and the k-point coordinates and the target is the energy dispersion $E_{\text{k}}(\varepsilon)$ where $n$ is the band index, k is the wavevector, and $\varepsilon$ is the crystal strain tensor. The hidden-layer structures of the two associated deep NNs are also depicted. (B) Better bandgap-fitting results measured by MAE are yielded by data fusion compared with the sole use of $\varepsilon$ as input to fit GW data. (Inset) Data-fusion-based learning of the difference between $E_{\text{PBE}}$ and $E_{\text{GW}}$ ensemble methods on decision-tree classifiers including gradient boosting regression (GBR) and random forest regression (RFR), Lagrange interpolation and NN are adopted for ML fitting. (C) Reachable bandgap values for various $h$ within the whole deformation space for silicon. The region where the strained silicon has a direct bandgap is colored in red. The circle at $h = 1.35$ meVÅ$^2$ indicates the lowest energy penalty for the semiconductor-to-metal transition. (D) Diamond bandgap envelope extending toward the small-bandgap semiconductor region. The upper- and lower-envelope functions are indicated by black and red dots, respectively. The arrows on the horizontal axes in C and D indicate reachable $h$ by the in situ experiments (7, 8).
In ESE experiments, the objective is to identify the highest or lowest bandgap that can be achieved through the expenditure of a certain elastic strain energy density \( h \) defined as

\[
h(e) = \frac{E(e) - E^0}{V_0},
\]

where \( E(e) \) is the total energy of the cell deformed by strain \( e \), and \( E^0 \) and \( V^0 \) are the total energy and volume of the undeformed cell, respectively. Here, we data-mine the 6D deformation by ML, the bandgap distribution and the elastic strain energy density against \( e \). The many-to-many relation between \( h(e) \) and the bandgap \( E_g(e) \) is shown in Fig. 1. In the stress-free equilibrium state, silicon has a bandgap of 1.1 eV; with an increase in strain energy density, a variety of possible bandgaps emerge. Even silicon with as little strain energy density as 0.2 meV/Å\(^3\) can become quite a different material from the stress-free silicon. As \( h \) further increases, the largest allowable bandgap drops and an “envelope” forms, as evidenced by the change of maximal and minimal bandgap reachable under a fixed \( h \). The shading of the envelope regions in Fig. 1C and D reflects the distribution of the available bandgap. A darker shading qualitatively indicates that the amount of possible strains to achieve a specific bandgap at a given \( h \) is higher. Outside the envelope the shading color is white, meaning that the corresponding bandgap is not attainable. Mathematically, we can define the cumulative “density of states” of bandgap as

\[
c(E_g; h') = \int_{h(h')} d^6 e \delta(E_g' - E_g(e))
\]

where \( d^6 e = de_1de_2de_3de_4de_5de_6 \) in the 6D strain space, \( \delta(\cdot) \) is the Dirac delta function, and \( H(\cdot) \) is the Heaviside step function. We then define the density of states of bandgap (DOB) at \( h' \) by taking the derivative of \( c(E_g; h') \) with respect to \( h' \):

\[
\rho(E_g; h') = \frac{dc(E_g; h')}{dh'} = \int d^6 e \delta(E_g' - E_g(e)) \delta(h' - h(e)).
\]

The meaning of DOB can be described by considering all possible elastically strained states within the \( [h - \frac{dh}{2}, h + \frac{dh}{2}] \) energy interval, and the resultant distribution of bandgaps arising from these states. The DOB function \( \rho(E_g; h) \) offers a blueprint for determining which bandgaps are accessible at what energy cost. One can use the definition (3) not only for the electronic bandgap, but also generally for any scalar property that will provide an easy-to-visualize map for deep ESE such as the thermoelectric figure of merit \( zT \), Baliga’s figure of merit (14), Curie temperature, etc. (4). An upper-envelope function \( E_{\text{upper}}(h) \) and lower-envelope function \( E_{\text{lower}}(h) \) can also be defined based on \( \rho(E_g; h) \):

\[
E_{\text{upper}}(h) = \max \{ \rho(E_g; h) \},
\]

\[
E_{\text{lower}}(h) = \min \{ \rho(E_g; h) \},
\]

which are rendered as black and red dotted lines in Fig. 1C and \( D \), so the nonzero DOB falls within \( (E_{\text{lower}}(h), E_{\text{upper}}(h)) \). In deep ESE, \( E_{\text{lower}}(h) \) also indicates the path to obtain the fastest change in \( E_g \). For instance, if the goal is to reduce the bandgap of silicon from 1.1 eV as fast as possible, with the least cost of elastic energy, the red-dotted line in Fig. 1C (which is further detailed in Fig. 2A) \( E_{\text{lower}}(h) \) offers the best design of the strain tensor \( e \) to achieve this goal.

It is seen from Fig. 1C and \( D \) that, with the application of a relatively small amount of mechanical energy, the overall distribution of Si bandgap shifts downward. This means that by modulating the tensorial strain (shear/tension/compression combinations) in multiple directions, strained silicon becomes capable of absorbing a different part of the electromagnetic spectrum than when it is in a stress-free state. It was also found that at 1.35 meV/Å\(^3\) the bandgap of Si can vanish, corresponding to the minimum energy required for semiconductor-to-metal transition in the whole 6D strain space (see Fig. 2B for the band structure, which corresponds to the red circle in Fig. 1C). Fig. 2A further illustrates that silicon’s “most energy efficient path to metallization” is actually a curved path in the strain space: The initial fastest-descent direction for \( E_g \) (at \( h = 0 \)) is quite different from when \( E_g \) hits zero at \( h = 1.35 \) meV/Å\(^3\) and thus linear perturbation theory such as the deformation potential theory (15) is not expected to work well in deep-strain space. It is not straightforward yet to achieve this complex optimal strain state in 6D experimentally, despite Feynman’s prophecy to use “a hundred tiny hands” (3). To provide experimental guidance, we further implemented our ML model in experimentally feasible uniaxial strain cases. It is found that (111) crystal direction is the most energy efficient uniaxial strain direction for Si bandgap engineering (SI Appendix, Fig. S3). A complete ranking of the common crystal directions in terms of their ability to lower Si bandgap can be found in SI Appendix, Note S3. In the case of diamond, deep ESE provides an opportunity to reduce its bandgap to a level comparable to that of InAs. Our results thus demonstrate that by straining diamond in the most optimal way, it can be transformed to mimic the properties of a lower-bandgap semiconductor while almost preserving its own uniqueness such as high strength and thermal conductivity, thereby

Table 1. Root-mean-squared error for various ML algorithms for the bandgap and band structure prediction tasks from PBE data for silicon (in units of electron volts)

<table>
<thead>
<tr>
<th>ML input</th>
<th>GBR</th>
<th>RFR</th>
<th>NN</th>
<th>ML target</th>
</tr>
</thead>
<tbody>
<tr>
<td>e^{3D}</td>
<td>0.0367</td>
<td>0.0247</td>
<td>0.0049</td>
<td>Bandgap</td>
</tr>
<tr>
<td>e^{4D}</td>
<td>0.0743</td>
<td>0.0781</td>
<td>0.0264</td>
<td>Bandgap</td>
</tr>
<tr>
<td>k and e^{VB}</td>
<td>0.1125</td>
<td>0.1078</td>
<td>0.0131</td>
<td>( \varepsilon_s(k; e) )</td>
</tr>
<tr>
<td>k and e^{CB}</td>
<td>0.1593</td>
<td>0.1555</td>
<td>0.0184</td>
<td>( \varepsilon_s(k; e) )</td>
</tr>
</tbody>
</table>

\( e^{3D} \) and \( e^{4D} \) denote three-normal-strains deformation and general deformation cases, respectively. For all of the details on ML and DFT methodology, optimization, and implementation, see Methods and SI Appendix, Notes S1 and S2 and Figs. S1 and S2.

Fig. 2. (A) The most energy-efficient strain pathway to reach the zero-bandgap state, i.e., the lower-envelope function \( E_{\text{lower}}(h) \) in silicon corresponding to the red-dotted line in Fig. 1C. The zero-bandgap state (open red circle on the horizontal axis of Fig. 1C) corresponds to the deformation case of \( \varepsilon_1 = 0.5522\% \), \( \varepsilon_2 = -1.2582\% \), \( \varepsilon_3 = -0.138\% \), \( \varepsilon_4 = -1.9168\% \), \( \varepsilon_5 = 0.7411\% \), and \( \varepsilon_6 = 1.6878\% \). (B) GW band structure associated with this deformation. The fractional coordinates for the three high-symmetry points along the selected \( k \) path are \((0.5, 0, 0), (0, 0, 0), \) and \((0.5, 0, 0.5)\), respectively.
paving the way for designing hitherto unexplored combinations of material characteristics.

Another important issue for optical applications pertains to whether the bandgap is direct or indirect. This direct bandgap envelope is a subset of DOB. We define the density of direct bandgaps (DOD) in parallel to [2]–[4], but with $E_{\text{direct}}^g$ instead of $E_g^2$ to obtain DOD $\rho_D(E_{\text{direct}}^g, h)$ and its bounds $E_{\text{upper}}^g(h)$, $E_{\text{lower}}^g(h)$. Obviously, if direct bandgaps exist at any strain, for that strain there will be

$$\left(E_{\text{lower}}^g(h), E_{\text{upper}}^g(h)\right) \subseteq \left(E^g_{\text{direct}}(h), E^g_{\text{upper}}(h)\right). \quad [5]$$

Our deep ESE model found within experimentally accessible strain range that the indirect-to-direct bandgap transition takes place in silicon in the high-$h$ region and a minimum strain energy density $h_{\text{d}}^{\text{min}}$ around 15.4 meV/Å$^3$ exists for the direct bandgap to appear (the red region in Fig. 1C):

$$h_{\text{d}}^{\text{min}} = \min \sup_{h} \left(E_{\text{upper}}^g(h) - E_{\text{lower}}^g(h)\right). \quad [6]$$

This little “island” of DOD within the ocean of DOB can be achieved by applying $e_1 = e_2 = e_3 \geq 9.3\%$.

The conventional way to modulate electronic properties in semiconductors is the so-called compositional grading technique. Through varying the stoichiometry of an alloy semiconductor, as for example by molecular beam epitaxy, a graded bandgap can be produced (16). This method of tweaking the material property is conceptually based on chemical alloying, whereby the chemical composition is tuned in an alloy melt to produce desirable strength or ductility. Invoking this approach, conventional bandgap engineering resorted to chemical alloying such as GaAl$_x$As$_{1-x}$ or Ga$_{1-x}$In$_x$As (17). However, we have demonstrated here that the stress-free situation is usually not the optimal state for a figure of merit, and elastic strains allow the bandgap to exhibit many more possible values so that each pure material candidate should occupy a much larger hyperspace enabled through the achievable 6D strain space. The more general bandgap engineering approach could utilize gradients in both composition and strain to achieve the desired band alignment.

Exploring Bandgap Ridgelines in Strain Space. Here we choose the most widely used semiconductor material, Si, as an example to demonstrate the generality and flexibility of our method. Since the full 6D strain space does not allow for easy visualization, we restrict ourselves to tensile and compressive normal strains only ($E_4 = E_5 = E_6 = 0$) for illustration purposes. Note that combinations of tensile and compressive strains can be used to generate shear strains in the material even though not all shear strains are considered. Fig. 3A illustrates the isosurface for Si bandgap, i.e., the set of points in the strain space where the bandgap equals some given value, for different $E_g$ levels obtained by our high-throughput NN model. The most striking visual feature of this $E_g$ isosurface in $e_1e_2e_3$ space is its piecewise smoothness. There are cusps singularities of different order: ridgelines where two smooth pieces of the $E_g$ isosurface meet, and corners where three ridgelines meet. These singularities are characterized by discontinuities in the slope (but not value) of the isosurface in the strain space due to band cross-over or even band topology change. Such cusps features also exist in $E_g$ isosurface in the general-$e_1e_2e_3e_4e_5e_6$ space, although they are more difficult to visualize directly. One can mathematically define these nonsmooth features on the 5D isosurface (embedded in 6D) as nth-order ridges ($E_g$) if they are differentiable in 5-n directions, while sustaining a change in slope in the other n directions in the strain space.

Since both the crystal structure and deformation tensor have symmetries, and the bandgap as a function of strain is invariant with respect to some of them, the “paleolith”-like $E_g$ isosurface (in analogy to the Tresca yield surface in strength of materials) has the following symmetry structure:

![Fig. 3](image-url)
i) The points \( \mu \) (the most “compressive” hydrostatic strain point on the \( E_g \) isosurface) and \( \chi \) (most “tensile” hydrostatic strain point on the \( E_g \) isosurface) lie on the \( e_1 = e_2 = e_3 \) line. We thus denote their strain-space coordinates by \((a,a,a)\) and \((b,b,b)\), respectively. At small or moderate \( E_g \), \( \mu \) splits and gives rise to a topologically triangular region \( A(1,2,3) \) as shown in Fig. 3A. It will later be shown these \( \chi \)-type points form the direct bandgap region on the \( E_g \) isosurface.

ii) The points \( \alpha_j (j=1,2,3) \) form a regular triangle which lies in a plane orthogonal to the \( e_1 = e_2 = e_3 \) line. Their coordinates are denoted by \((c,d,d)\), \((d,c,d)\), and \((d,d,c)\), respectively.

iii) The points \( \beta_j (j=1,2,3) \) also form a regular triangle which lies in a plane orthogonal to the \( e_1 = e_2 = e_3 \) line. Their coordinates are denoted by \((f,e,e)\), \((e,f,e)\), and \((e,e,f)\), respectively.

The shape of the isosurface is similar for both PBE and GW bandgaps, although the specific strain values may differ for the same PBE and GW bandgap levels. It was found that the easiest way [with the least \( h(\mathbf{E}) \)] to obtain the 0-eV bandgap without any shear strain is to apply a normal strain of \(-3.86\) and \(4.36\)% along any two of the three \((100)\) directions while leaving the third \((100)\) direction undeformed. Therefore, there are six strain cases that are equivalent, as indicated by red dots in Fig. 3C. The position of the vertices of the \( E_g \) isosurface in the strain space is the function of selected bandgap value, and the detailed relationship between the bandgap and the strains is shown in Fig. 3D. According to our PBE + GW model, the maximum bandgap reachable by strained silicon is 1.24 eV under a hydrostatic tensile strain of 6.5%. It should be noted that silicon strained to such an extent can nearly reach the maximum theoretical efficiency, known as the Shockley–Queisser limit (18), of a single p–n junction solar cell, demonstrating possible application of ESE in solar energy conversion devices.

The formation of the \( E_g \) isosurfaces, such as the ones in Fig. 3A, is due to the relative position of the valence band maximum (VBM) and the conduction band minimum (CBM). Despite different shape variations of the two energy bands, modulating elastic strain provides possibilities for the VBM and CBM to differ by the same amount with respect to the vacuum level. For undeformed silicon with a bandgap of 1.1 eV, the VBM is located at the \( \Gamma \) point and the CBM lies on the straight line (the \( \Delta \) line) in the \( k \) space and is positioned at about 85% of the way from the Brillouin zone center to the zone boundary (19). Under 3D deformation, the cubic crystal symmetry of Si is lifted and we follow the \( k \)-point labeling scheme explained in SI Appendix, Note S1 and Fig. 1 to describe band extrema positions. It is found that VBM remains at \( \Gamma \) irrespective of deformation whereas the position of CBM can be greatly affected by external strains. Using the geometry of the \( E_g \) isosurface as a visualization tool, we identify four types of \( k \)-space transition in CBM that may happen across the ridgelines on the isosurface.

Starting with the strain points on the lower faces separated by \( \mu - \alpha_1 \) ridgelines of the \( E_g \) isosurface in Fig. 3A, we found that the CBM retains roughly the same relative position along the “\( \alpha \)”-type line as in the undeformed case, and that crossing the ridgelines only switches CBM among \( A = (0,k,0,k) \), \( \Delta = (k_1,0,k_1) \), and \( \Delta = (k_1,k_1,0) \), where \( k_1 \approx 0.425 \). In other words, \( \mu - \alpha_1 \), ridgeline corresponds to \( \Delta - \alpha_1 \) transition, \( \mu - \alpha_2 \) ridgeline corresponds to \( \Delta - \alpha_3 \) transition, \( \mu - \alpha_3 \) ridgeline corresponds to \( \Delta - \alpha_2 \) transition, and we can indeed label each carapace by its CBM character \( \Delta, \Delta, \Delta \). We term this transition occurring in the small strain region as the \( \Delta \) switching. In this case, the linear deformation potential theory can be used to describe the strain effects on the band extremum (15). However, investigation of the large deformation points on its upper faces in Fig. 3A reveals that the CBM would not retain its location and major changes would happen.

Our ML model captures the occurrence of “\( L - \Delta \)” transition across the \( \beta_1 - \alpha_1 \) ridgelines where the CBM changes to “\( L \)” points in \( k \) space: \( L_1 = (0.5, 0, 0) \), \( L_2 = (0, 0.5, 0) \), \( L_3 = (0, 0, 0.5) \); see Fig. 4A and B, where for example, “\( \Delta \)” carapace changes to “\( L_1 \)” carapace across the \( \alpha_1 - \beta_1 \) ridgeline, and “\( \Delta \)” carapace changes to “\( L_2 \)” carapace across the \( \alpha_1 - \beta_1 \) ridgeline. None of the ridgelines or carapaces (e.g., \( \Delta \) carapace bound by \( \mu - \alpha_1 - \beta_1 - \mu_2 - \mu_3 - \mu_2 \)) are truly flat. The large, nonperturbative deformation makes the conventional theory ineffective in predicting it. Moving further toward \( \Delta \) in the strain space, CBM would remain at \( L \) and a cross-over of the \( \Delta \) - \( \beta \) ridgelines is referred to as an \( L \) switching. Indirect-to-direct bandgap transition occurs near the upper tip of the paleolith-like isosurface where CBM appears at \( \Gamma \), as shown in Fig. 4C. This can be explained by the competition between drops of different band edges. In general, as strain increases, the band edge at both \( \Gamma \) and \( L \) would decrease. As a result of high strains, the energy decrease at \( \Gamma \) is faster and eventually the bandgap becomes direct, as shown in Fig. 4D. In this case, we transition for example from the \( L_4 \) carapace (\( \alpha_1 - \beta_1 - L_3 - \beta_1 - \alpha_1 \)) to “\( L \)” carapace (\( \alpha_1 - \beta_1 - L_3 - \beta_1 - \alpha_1 \)) across the \( \chi \)-fold \( \beta \) ridgeline. When the strained Si turns into a direct-bandgap semiconductor, it would exhibit a significant enhancement in its optical transitions around the fundamental adsorption edge compared with an undeformed Si, due to the elimination of phonon involvement to facilitate adsorption or emission. As absorbance increases exponentially with thickness in a material, a solar cell based on direct bandgap Si with high adsorption coefficient would require much less thickness to absorb the same amount of light, paving the way for the design of lightweight high-efficiency solar cells. SI Appendix, Table S2 summarizes all of the details of the \( k \)-space transitions, thus resolving the conduction band properties exhaustively for a wide range of strains.

Incremental Fitting. We next show that our NN-based surrogate models can successfully learn from several datasets and assimilate them. This capability is becoming increasingly important with the spread of materials property databases that collect data from different studies (20). The incremental training of the NN starts from the same weights but is done on the extended dataset with the additional data included. We also increase the learning rate of stochastic gradient descent algorithm and regularizers (dropout rate and weight regularization) to circumvent limitations arising from the same local minima of the loss function established during the training on the initial dataset. This allows the model to
not only handle additional training on the incoming data appended to a database but to do it much faster than from scratch.

Numerical experiments conducted on the NN model demonstrate that incremental fitting of the models effectively reduces the error on a new dataset, see SI Appendix, Table S3. Such incrementally fitted models are, thus, equally applicable to the bandgap approximation and various optimization tasks. Moreover, these models may be reused when shifting to other materials such as Ge, since the implicit insights about symmetries, transitions, and extreme cases are stored in the parameters of NN. Training the model for the other material starting from the weights for Si would significantly reduce the time and amount of data needed due to knowledge transfer, also referred to as transfer learning (21), leading to rapid development of versatile surrogate models for ESE.

Discussion

ML models provide an efficient way of representing electronic band structure allowing for studies and accurate ESE predictions of a variety of physical phenomena such as band warping, degeneracy lifting, indirect-to-direct bandgap transition, and semiconductor-to-metal transition. In previous studies, bandgap engineering was conducted largely by tuning only one or two strain components. Our ML methods are capable of exploring the full spectrum of possibilities by efficiently analyzing highly nonlinear relations between electronic band dispersion and the strain tensor. To this end, the electronic band structure of silicon is accurately captured from ML through only a limited amount of calculations. Employing deep-NN algorithms, the bandgap of Si can be fitted as a function of strain within milli-electron-volt accuracy.

In prior approaches of analytically describing strain effects by traditional means, the linear deformation potential theory has often been invoked and its insufficiency at large deformation cases (24) makes it impossible to map out the entire strain space. By contrast, the general and systematic ML framework we demonstrate here makes the problem of representing the bandgap, and more broadly, the band structure, as a function of 6D strain computationally tractable. Many avenues remain for the application of our models on multiple fronts. Among these we mention the extension of the model to increasingly complex material structures, predicting their bandgap and band structure, and phonon and photonic band structure.

Different strains may result in the same bandgap, and in seeking a specific bandgap, or any other materials figure of merit, one should choose the strain with a minimal effort required given the nonuniqueness of choice of a given target property or figure of merit. For this purpose, the DOB envelope we developed here is essential in understanding and fully utilizing deep ESE. In our work, we use the elastic strain energy density as a scalar metric or “norm” of the strain tensor for rationally choosing the ESE route that requires the least energy metastability and corresponds to the safest deformation manner in principle. For example, we have demonstrated that our model is able to locate the most energy-efficient pathway in the entire strain space to transform silicon from a semiconductor to a metal or to convert diamond from an ultrawide-bandgap material to a wide or even small-bandgap semiconductor. Latest advances in methods to apply large strains have included wide adoption of microelectromechanical systems and nanoelectromechanical systems, in situ indentation techniques, and nano-cantilever-beam bending (7, 8) and anviling (22) on materials across different size scales. The growing variety of technologies available to apply strains in a precisely controlled manner through mechanical, electrical, magnetic, thermal, and other means also promises the design of experiments to impose and tune different components of strains (23–26). Thanks to the expanding maturity of available tools, experimental implementation of the ESE approaches identified here for the 6D strain space is a next step in advancing further progress in this field. The distinctive ML model we propose here thus offers a potentially powerful method in guiding the design of approaches for a wide variety of semiconductor materials including silicon and diamond that could lead to performance improvement in applications as diverse as flexible electronics (27), nanomechanical resonators (28), optical fibers (23), and energy storage systems (29).

Methods

First-Principles Calculations. Details for DFT simulations are in SI Appendix, Note S2.

ML. NN and tree-based ensemble algorithms were adopted. More details are in SI Appendix, Note S2.

Data Fusion. Details for data fusion are in SI Appendix, Note S2.

ACKNOWLEDGMENTS. The authors thank Dr. Wenbin Li and Dr. Xiaohui Liu. The computation works were performed on supercomputers at Massachusetts Institute of Technology (MIT) and Skolkovo Institute of Science and Technology (Skolktech). S.S. acknowledges support from Nanyang Technological University through the Distinguished University Professorship. The work is supported by the Skolktech Next Generation Program 2016–TNGP (a Skolktech–MIT joint project).